The Accuracy of Density Forecasts from Foreign Exchange Options

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Abstract

Financial decision makers often consider the information in currency option valuations when making assessments about future exchange rates. The purpose of this paper is to systematically assess the quality of option based volatility and density forecasts. We use a unique dataset consisting of over 10 years of daily data on over-the-counter currency option prices. We find that the OTC implied volatilities provide largely unbiased and fairly accurate forecasts of 1-month and 3-month ahead realized volatility. Furthermore, we find that the 1-month option implied density forecasts are well calibrated for the centre of the distribution but we find evidence of misspecification in the tail density forecasts.

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Introduction

Financial decision makers often consider the forward-looking information in currency option valuations when making assessments about future developments in foreign exchange rates. Financial market monitoring by central banks and international supervisory agencies is also influenced by information implied in options valuations. Leading examples hereof include the Bank for International Settlements (2003), the Bank of England (2000), the International Monetary Fund (2002), and OECD (1999). Option implied at-the-money volatilities can be used as forecasts of realised volatility and interval and density forecasts can be extracted from strangles and risk-reversals. The purpose of this paper is to assess the quality of such volatility, interval and density forecasts. Our work is based on a unique database consisting of more than ten years of daily quotes on European currency options from the OTC market.
[1] The OTC quotes include at-the-money implied volatilities, strangles and risk-reversals on the dollar, yen and pound per euro[2] as well as on the yen per dollar. From this data we have constructed daily 1-month density and interval forecasts using the methodology in Malz (1997).

The main findings of the paper are as follows: First, using Mincer-Zarnowitz regressions we find that the OTC implied at-the-money volatilities provide essentially unbiased and fairly accurate forecasts of realized 1-month and 3-month volatility. Second, we find that the option-based density forecasts are rejected in general. Tests on subsets of the support of the distribution reveal that while the sources of rejections vary from currency to currency misspecification of the distribution tails is common. Third, matching the density forecasting results, we find that wide-range interval forecasts are often misspecified whereas narrow-range interval forecasts are well specified.

Our volatility forecasting results is related to several papers working with market traded options rather than OTC contracts that we use here. Beckers (1981) finds that not all available information is reflected in the current option price and question the efficiency of the option markets. Canina and Figlewski (1993) find implied volatility to be a poor forecast of subsequent realized volatility. Lamoureux and Lastrapes (1993) provide evidence against restrictions of option pricing models which assume that variance risk is not priced. Jorion (1995) finds that statistical models of volatility based on returns are outperformed by implied volatility forecasts even when the former are given the advantage of ex post in sample parameter estimation. He also finds evidence of bias. More recently, Christensen and Prabhala (1998) using longer time series and non overlapping data find that implied volatility outperforms past volatility in forecasting future volatility. Fleming (1998) finds that implied volatility dominates the historical volatility in terms of ex ante forecasting power and suggests that a linear model which corrects for the implied volatility's bias can provide a market-based estimator of conditional volatility. Blair, Poon, and Taylor (2001), find that nearly all relevant information is provided by the VIX index and there is not much incremental information in high-frequency index returns. Neely (2003) finds that econometric projections supplement implied volatility in out-of-sample forecasting and delta hedging. He also provides some explanations for the bias and inefficiency pointing to autocorrelation and measurement error in implied volatility. More recently, Pong, Shackleton, Taylor and Xu (2004) using OTC data obtain superior accuracy of the historical forecasts, relative to implied volatilities from the use of high frequency returns, for horizons up to one week. Covrig and Low (2003) use OTC data to find that quoted implied volatility subsumes the information content of historically based forecasts at shorter horizons, and the former is as good as the latter at longer horizons.

Our paper contributes in two areas of the literature. First, to our knowledge, the empirical performance of option-based interval and density forecasts has not been systematically explored so far. Second, while there is a considerable literature on implied volatility forecasts from market-traded options, OTC data have only recently been employed.

In addition to volatility forecasts we evaluate option-based interval and density forecasts which are widely used by financial institutions but which have not been systematically assessed so far. OTC options are quoted daily with fixed moneyness in contrast with market-traded options which have fixed strike prices and thus time-varying moneyness as the spot price changes. This time-varying moneyness complicates the use of market-traded options for interval and density forecasting in that the effective support of the distribution is changing over time.

The remainder of the paper is structured as follows. Section 2 defines the competing volatility forecasts we consider and applies the Mincer-Zarnowitz methodology for volatility forecast evaluation. Section 2 suggests methods for evaluating density forecasts from option prices and present results from these methods. Section 3 suggests a method for evaluating interval forecasts from option prices and present results from this method. Section 4 summarizes and discusses potential points for future research.

1. Volatility Forecast Evaluation

It is widely believed that conditional density dynamics in daily FX rates are mainly driven by conditional variance dynamics[3] and exploring these explicitly first is therefore sensible. So, while the main objective of this paper is to evaluate option implied densities, we first undertake a brief study of option implied volatility forecasts. In order to evaluate the informational content of the volatilities implied from currency options, we define the realized future volatility for the next *h* days to be

$$\sigma_{t,h}^{RV} = \sqrt{\frac{252}{h}\sum_{i=1}^{h} R_{t+i}^2}$$

in annualized terms, where $R_{t+i} = \ln(S_{t+i}/S_{t+i-1})$ is the FX spot return on day t+i.[4] This realized volatility will be our forecasting object of interest in this section. Notice that we implicitly assume that the FX returns have no autocorrelation. We have verified that this assumption is innocuous for the daily returns in highly liquid markets that we use.[5]

We will consider four competing forecasts of realized volatility. First and most importantly the implied volatility from at-the-money OTC currency options with maturity *h*, where *h* is either 1 month or 3 months corresponding to roughly 21 and 63 trading days respectively. Denote this options-implied volatility by $\sigma_{t,h}^{IV}$.

The other three volatility forecasts are derived from historical FX returns only. The simplest possible forecast is the historical *h*-day volatility, defined as $\sigma_{t,h}^{HV} = \sigma_{t-h,h}^{RV}$.

We can instead consider volatilities that apply an exponential weighting scheme putting progressively less weight on distant observations. The simplest such volatility is the Exponential Smoother or RiskMetrics (RM) volatility, where daily variance evolves as

$$\sigma_{t+1}^2 = \lambda\sigma_t^2 + (1-\lambda)R_t^2$$

Following JP Morgan we simply fix λ=0.94 for all the daily FX returns. The fact that the coefficients on past variance and past squared returns sum to one makes this model akin to a random walk in variance.

Finally we consider a simple, symmetric GARCH(1,1) heteroskedastic (GH) model, where the daily variance evolves as

$$\sigma_{t+1}^2 = \omega + \beta\sigma_t^2 + \alpha R_t^2$$

In contrast with the RiskMetrics model, the GARCH model implies a non-constant term structure of volatility which we use to calculate the multi-day (annualized) variance forecasts.[6]

Our dataset consists of daily FX rates from the BIS and daily European OTC implied FX volatilities from Citibank observed from March 31, 1992 to February 19, 2003. Prior to the euro introduction on January 1, 1999 we observe FX options denoted against the Deutschmark

(DEM). Thus, prior to January 1, 1999 we use DEM options to forecast DEM volatility and afterwards we use euro options to forecast euro volatility.[7] We consider four FX rates: euro (DEM) versus USD, JPY and GBP, as well as USD versus JPY. We will refer to these simply as "USD", "JPY", "GBP" and "JPY/USD" below.

Christensen, Hansen and Prabhala (2001) argue that, in terms of forecasting properties, OTC options data may be of superior quality relative to exchange traded options. This is because OTC prices are quoted daily with a fixed "moneyness". Moreover, the large trading volume in OTC could make the OTC quotes a potentially more reliable source for information extraction.

The Bank for International Settlements (2005) estimate that on December 31, 2004, the notional amount outstanding in the global OTC derivatives market was 248,288 billion US dollar. This compares to the notional amount outstanding in globally exchange-traded contracts which was 46,592 billion US dollar on the same date. By this measure the global OTC derivatives market is thus about five times larger than the exchange traded market.

We are now ready to assess the quality of the different volatility forecasts. This will be done using Mincer and Zarnowitz (1969) predictability regressions. We run four regressions per FX rate corresponding to one for each of the volatility forecasts

$$\sigma_{t,h}^{RV} = a_j + b_j \sigma_{t,h}^{j} + \varepsilon_{t,h}^{j}, \text{ for } j = \text{IV, HV, RM, GH}$$

These regressions are run for h=21 and 63 corresponding to the 1-month and 3-month option maturities. Table 1 reports the Mincer-Zarnowitz decomposition of the mean-squared-error (MSE) of the above regressions run on 1,044 consecutive trading days for each horizon. The MSE is decomposed into bias squared, inefficiency and residual variation calculated from

$$MSE^j = \left(E[\sigma_{t,h}^{RV}] - E[\sigma_{t,h}^{j}]\right)^2 + (1-b_j)^2 \operatorname{Var}(\sigma_{t,h}^{j}) + (1-R_j^2)\operatorname{Var}(\sigma_{t,h}^{RV})$$

where R^2 is the usual measure of regression fit. The decomposition in Table 1 is reported in percent of the MSE. The left panel gives the 1-month horizon result and the right panel the 3-month horizon result.

Perusing first the MSE columns we see that the option implied volatilities have the lowest MSE for all currencies and for both horizons, with the only exception being the 3-month USD where the GARCH forecast is slightly better than IV. Considering next the residual variation column (the right-most column I n each panel) we see that six out of eight cases, the option implied volatility forecast has the highest relative residual variation, meaning that a relatively small amount of its MSE comes from systematic errors in the form of bias and inefficiency. The "inefficiency" column reveals that the option implied volatilities have the lowest relative inefficiency in five out of eight cases and it is small in the remaining three cases as well. Looking finally at the squared bias column we see that the relative bias is small in all four forecasts but it is never the smallest for the option implied forecasts. In sum, the option implied forecasts have the lowest MSE with a low degree of systematic error, but a slight bias.

Bollerslev and Zhou (2005)[8] point out that if the volatility risk is priced in the options markets then we should expect to find a bias in the implied volatility forecasts. In a standard stochastic volatility set up, it can be shown that if the price of volatility risk is zero, the process followed by the volatility is identical under the objective and the risk neutral measures. In such a case there would be no bias. However, the volatility risk premium is generally estimated to be negative which in turn implies that the volatility process under the risk neutral measure will have higher drift. This is also consistent with the fact that implied volatilities are empirically found to be upward biased estimates of the objective volatility. Nevertheless, the results in Table 1 indicate that the bias in the option implied volatility forecasts in our application constitutes a relatively minor part of the total forecast MSE.

The empirical properties of the option implied volatility forecasts found above suggest that density forecasting based on the implied information in option prices holds promise. This is the topic to which we now turn.

2. Option Implied Density Forecasts

The information in currency options can be used not only for volatility forecasting but for spot rate density forecasting more generally. The question we address in this section is: How well do risk neutral densities computed via a widely used methodology describe the physical distribution of the future FX spot rate? We fully recognize that theoretically the risk neutral and physical distributions can differ, but financial decision makers often consider risk neutral densities (see e.g. Bank of England, 2000, Bank for International Settlements, 2003, OECD, 1999, and International Monetary Fund, 2002) when making assessments about future exchange rates.

The pragmatic approach taken can be justified by considering that for currencies the risk premium, i.e. the conditional mean, which would largely determine the difference between the risk neutral and physical densities, may not be as important for the density shape as the higher order moments in particular the conditional variance. Moreover, at the one month horizon the magnitude of the drift is likely to be small compared with the dispersion. The conditional variance in turn may not be too different in the risk neutral versus physical distributions. The volatility forecasting results in Table 1 above indeed suggests that this is the case.

An option implied density forecast could certainly be misspecified due to the presence of a currency risk premium. But rejection could also come from methodological and/or data problems in the construction of the density forecasts. For someone wishing to use option implied density forecasts of the FX spot rate going forward, any source of error is important. The tests we consider below are designed to capture misspecification in general. Before turning to the main topic of density forecast evaluation, we give a short overview of the construction of the option implied density forecasts.

2.1 Option Implied Density Forecast Construction

When constructing the option implied density forecasts we rely on the methodology in Malz (1997) which is tailored to the kind of OTC data available to us. Each day we observe the implied volatility of three one-month derivatives: An at-the-money (ATM) call, a strangle (STR), and a risk reversal (RR).[9] The ATM call has a delta[10] of 0.5 by definition and the strangle and risk reversals are quoted with strike prices corresponding to deltas of .25 and .75. Writing the implied volatility for each contract as a function of delta σ(δ) yields

$$\begin{aligned}\sigma_{ATM} &= \sigma(0.5)\\ \sigma_{RR} &= \sigma(0.25)-\sigma(0.75)\\ \sigma_{STR} &= 0.5([\sigma(0.25)-\sigma_{ATM}]-[\sigma_{ATM}-\sigma(0.75)])\end{aligned}$$

Notice that the risk-reversal and strangle volatilities can be viewed as (proportional to) discrete approximations of the first and second derivative of an implied volatility function of delta. This in turn suggests approximating the implied volatility function using a second order Taylor expansion

$$\sigma(\delta) = b_0\sigma_{ATM} + b_1\sigma_{RR}(\delta-0.5) + b_2\sigma_{STR}(\delta-0.5)^2$$

Ensuring that the implied volatility function fits exactly at the three observed deltas $\{0.25,0.5,0.75\}$ gives three equations in $\{b_0, b_1, b_2\}$ which when solved yield

$$\sigma(\delta) = \sigma_{ATM} - 2\sigma_{RR}(\delta-0.5) + 16\sigma_{STR}(\delta-0.5)^2$$

Breeden and Litzenberger (1978) provides the necessary link between the implied volatility function and the risk-neutral density function. They show that the second derivative of the call option price, *c*, with respect to the strike price, *K*, is proportional to the risk-neutral density, *f*(*), of the underlying asset

$$\frac{\partial^2 c}{\partial K^2} = \exp(-r\tau) f(K)$$

where r denotes the risk-free rate and τ the time to maturity of the option which is one month in our application. Malz (1997) approximates *f*(*) by calculating the numerical second-derivative of the Black-Scholes call price allowing for the quadratic implied volatility function above.

While the Malz (1997) methodology is only one of many available, we use it because it is explicitly tailored to the data we have and because other methods typically require more observations per day. Interesting recent contributions in this area include Ait-Sahalia and Duarte (2003) and Ioffe (2004). We are also motivated to use Malz's method by the observation that many large institutions (see e.g. Bank of England, 2000, Bank for International Settlements, 2003, OECD, 1999, and International Monetary Fund, 2002) indeed use this methodology.

Having constructed daily density forecast for the 30-day-ahead FX spot rate, we now turn to the topic of density forecast evaluation.[11]

2.2 Probability Transform Variables

Let $F_{t,h}(S)$ and $f_{t,h}(S)$ denote the cumulative and probability density function forecasts made on day *t* for the FX spot rate on day t+h using the Malz (1997) methodology described above. We can then define the so-called probability transform variable as

$$U_{t,h} \equiv \int_{-\infty}^{S_{t+h}} f_{t,h}(u)du \equiv F_{t,h}(S_{t+h}).$$

The transform variable captures the probability of obtaining a spot rate lower than the realization where the probability is calculated using the density forecast. As it is interpretable as a probability it can only take on values in the interval [0,1]. Notice that if the density forecast is correctly calibrated then we should not be able to predict the value of the probability transform variable $U_{t,h}$ using information available at time *t*. In other words we should not be able to forecast the probability of getting a value smaller than the realization. Thus, if the density forecast is a good forecast of the true probability distribution then the corresponding transform variable will be distributed as an independent uniform variable on the [0,1] interval.

Consider an extreme counterexample where all the realizations of the transform variable $U_{t,h}$ fall between 0 and 0.5. In this case all the realizations of the forecasted variable $S_{t,h}$ fall in the left side of the distribution forecast which in turn implies that the left side is too likely and the right side is too unlikely in the density forecast. Observations should fall across the entire range of the forecasted density and with likelihood equal to the probability specified in the density forecast. Otherwise the density forecast is misspecified and the probability transform variable will not be uniform. In the time series context where the density forecasts are conditional on information available at time *t* and thus varying over time, the transform variable should be not only uniform but i.i.d. uniform. We should not be able to predict the realization of the transform variable with information available at time *t*.

Figure 1 assesses the unconditional distribution of the probability transform variable $U_{t,h}$ for each spot rate through a simple histogram. If the density forecast is correctly calibrated then each of the histograms should be roughly flat and a random 10% of the 31 bars should fall outside the two horizontal lines delimiting the 90% confidence band.

It appears that the histograms display certain systematic differences from the uniform distribution. Notice in particular that the JPY/EUR histogram (top right panel) shows a systematically declining shape moving from left to right. This is indicative of the forecasted mean spot rate being wrong. There are too many observations where the realized spot rate lies in the left side of the forecasted distribution (and generates a $U_{t,h}$ less than 0.5) and vice versa. In the USD/EUR case (top left panel) it appears that there are not enough observations in the two extremes, which suggests that the forecasted density has tails, which are too fat. Finally, the JPY/USD distribution (bottom right panel) appears to be misspecified in the right tail.

For the purpose of statistical testing it is more convenient to work with normally distributed rather than uniform variables for which the bounded support may cause technical difficulties. As suggested by Berkowitz (2001)[12] we can use the standard normal inverse cumulative density function to transform the uniform probability transform to a normal transform variable

$$Z_{t,h} = \Phi^{-1}\left(U_{t,h}\right) = \Phi^{-1}\left(F_{t,h}\left(S_{t+h}\right)\right)$$

If the implied density provides an accurate forecast of the physical density, it must be the case that the distribution of $U_{t,h}$ is uniformly distributed and independent of any variable X_t observed at time t. Consequently the normal transform variable $Z_{t,h}$ must be normally distributed and also independent of all variables observed at time t.

Figure 2 assesses the unconditional normality of the normal transforms by plotting the histograms with a normal distribution superimposed.[13] The normal histograms typically confirm the findings in Figure 1 but also add new insights. While it appeared in Figure 1 that the GBP/EUR had fairly random deviations from the uniform distribution, it now appears that the normal transform is systematically skewed compared with the superimposed normal distribution.

While the graphical evidence in Figures 1 and 2 is quite informative of the potential deficiencies in the option implied density forecasts, it may be interesting to formally test the hypothesis of the normal transforms following the standard normal distribution. We do this below.

2.3 Tests of the Unconditional Normal Distribution

We first want to test the simple hypothesis that the normal transform variables are unconditionally normally distributed. Basically, we want to test if the histograms in Figure 2 are significantly different from the superimposed normal distribution. The unconditional normal hypothesis can be tested using the first four moment conditions

$$E\left[Z_{t,h}\right]=0, \quad E\left[Z_{t,h}^2\right]=1, \quad E\left[Z_{t,h}^3\right]=0, \quad E\left[Z_{t,h}^4\right]=3$$

We still need to allow for autocorrelation arising from the overlap in the data and so we estimate the following simply system of regressions

$$\begin{aligned} Z_{t,h} &= a_1 + \varepsilon_{t,h}^{(1)} \\ Z_{t,h}^2 - 1 &= a_2 + \varepsilon_{t,h}^{(2)} \\ Z_{t,h}^3 &= a_3 + \varepsilon_{t,h}^{(3)} \\ Z_{t,h}^4 - 3 &= a_4 + \varepsilon_{t,h}^{(4)} \end{aligned}$$

using GMM and test that each coefficient is zero individually as well as the joint test that they are all zero jointly.[14] In each case we allow for 21 day overlap in the daily observations. The results of these tests are reported in Table 2.A which reports coefficient estimates along with *t*-statistics calculated using GMM with a quadratic kernel and a 21-day bandwidth.

Table 2.A shows that while only a few of the individual moments of the normal transform variable are found to be significantly different from the normal distribution, the joint (Wald) test that all moments match the normal distribution is rejected strongly in three cases and weakly in the case of the JPY/USD. We thus find fairly strong evidence overall to reject the option-implied density forecasts using simple unconditional tests.

In order to focus attention on the performance of the density forecasts in the tails of the distribution, we report QQ-plots of the normal transform variables in Figure 3. QQ-plots display the empirical quantile of the observed normal transform variable against the theoretical quantile from the normal distribution. If the distribution of the normal transform is truly normal then the QQ-plot should be close to the 45-degree line.

Figure 3 shows that the left tail is fit poorly in the case of the dollar, and that the right tail is fit poorly in the case of the pound and the JPY/USD. In the case of the dollar there are too few small observations in the data, which is evidence that the option implied density has a left tail that is too thick. The pound has too many large observations indicating that the right tail of the density forecast is too thin. In the JPY/USD case the right tail appears to be too thick. These findings are also evident from Figure 1.

Rejecting the unconditional normality of the normal transform variables is important, but it does not offer much constructive input into how the option-implied density forecasts can be improved upon. The conditional normal distribution testing we turn to now is more helpful in this regard.

2.4 Tests of the Conditional Normal Distribution

We would like to know why the densities are rejected, and specifically if the construction of the densities from the options data can be improved somehow. To this end we want to conduct tests of the *conditional* distribution of the normal transform variable. Is it possible to predict the realization of the time t+h normal transform variable using information available at time t? If so then this information is not used optimally in the construction of the density forecast.

The conditional hypothesis can be tested using the generic moment conditions

$$E\left[Z_{t,h} f_1(X_t)\right]=0, \quad E\left[Z_{t,h}^2 f_2(X_t)\right]=1, \quad E\left[Z_{t,h}^3 f_3(X_t)\right]=0, \quad E\left[Z_{t,h}^4 f_4(X_t)\right]=3$$

These moments are of course very general. As is often the case, we do not have much in terms of economic theory to guide us in the choice of moment functions information variables, X_t. Restricting attention to linear moment functions and variables already available to us, we implement the moments in a simple regression setup as follows

$$\begin{aligned}
Z_{t,h} &= a_1 + b_{11} Z_{t-h,h} + b_{12}\sigma_t^{IV} + \varepsilon_{t,h}^{(1)} \\
Z_{t,h}^2 - 1 &= a_2 + b_{21} Z_{t-h,h}^2 + b_{22}\left(\sigma_t^{IV}\right)^2 + \varepsilon_{t,h}^{(2)} \\
Z_{t,h}^3 &= a_3 + b_{31} Z_{t-h,h}^3 + b_{32}\left(\sigma_t^{IV}\right)^3 + \varepsilon_{t,h}^{(3)} \\
Z_{t,h}^4 - 3 &= a_4 + b_{41} Z_{t-h,h}^4 + b_{42}\left(\sigma_t^{IV}\right)^4 + \varepsilon_{t,h}^{(4)}
\end{aligned}$$

where we include the lagged power of the normal transform as well as the power of the current implied volatility as regressors. We can now test that the regression coefficients are zero.[15]

Table 2.B shows the estimation results of the regression systems for the four exchange rates. In line with previous results we find that the information in the implied volatility is not

used optimally in the construction of the option-implied density forecast for the GBP/EUR.[16] The lagged normal transform variable is significant in three out of four equations for the dollar, and in two equations for the yen. Table 2.B also shows that the Wald test of all coefficients being joint zero is strongly rejected for all four FX rates. It would therefore seem possible in general to improve upon the option-implied density forecasts studied here.

2.5 Piecewise Density Evaluation

The inspection of the uniform histograms reveals that the densities have problems particularly in the tails. In order formally test for misspecifications in the tail of the density forecast we extend upon the testing methodology proposed by Berkowitz (2001).

Let a, b $\in$ {0, 1} with $a \leq b$. From the properties of the uniform distribution it follows that if the density forecast is well specified then the collection of $U_{t,h} \in \{a, b\}$ will be uniformly distributed on {a, b} as well. We can than define another random variable $Y_{t,h} = (U_{t,h}-a)/(b-a)$ with $U_{t,h} \in \{a, b\}$ which is uniformly distributed on the interval {0, 1}. We can then use the inverse normal transformation again to test for the specification of the density forecast in the {a,b} interval.

The condition that $Y_{t,h}$ is uniformly distributed is necessary but not sufficient. For instance, $U_{t,h}$ could be uniformly distributed on a particular subset {a, b} of the interval {0, 1} but there could still be too few or too many observations falling in the interval {a, b}. A further necessary condition is that the coverage is correct. This corresponds to the requirement that the proportion of the observations falling in the {a, b} interval is equal to b-a.

This requirement can be translated into a condition, which can be tested in a GMM framework in addition to moment tests for the normality of $Z_{t,h} = \Phi^{-1}(Y_{t,h})$. To do so we define

$$I_{t,h} = \begin{cases} 1, \text{ if } U_{t+h} \in \{a,b\} \\ 0, \text{ if not} \end{cases}$$

And consider the following moments

$$\begin{aligned} &E\left[Z_{t,h} f_1(X_t)\right] = 0, \quad E\left[Z_{t,h}^2 f_2(X_t)\right] = 1 \\ &E\left[Z_{t,h}^3 f_3(X_t)\right] = 0, \quad E\left[Z_{t,h}^4 f_4(X_t)\right] = 3 \\ &E\left[I_{t,h}\right] = b - a \end{aligned}$$

Choosing particular moment functions and variables these conditions can be implemented in a regression system setup as follows. For the unconditional case we have

$$Z_{t,h} = a_1 + \varepsilon_{t,h}^{(1)}$$
$$Z_{t,h}^2 - 1 = a_2 + \varepsilon_{t,h}^{(2)}$$
$$Z_{t,h}^3 = a_3 + \varepsilon_{t,h}^{(3)}$$
$$Z_{t,h}^4 - 3 = a_4 + \varepsilon_{t,h}^{(4)}$$
$$I_{t,h} = \frac{\exp(a_5)}{1+\exp(a_5)} + \varepsilon_{t,h}^{(5)}$$

The estimation of the GMM system is done in such way as to specify the last condition as a logistic regression. The joint null hypothesis can be tested with a Wald test that $a_1 = a_2 = a_3 = a_4 = 0$, and $\exp(a_5)/(1+\exp(a_5)) = (b-a)$. The conditional test mirrors exactly the test for the entire distribution with the only addition of the fifth equation.

We implement the test on three subsets that are particularly relevant to our investigation: the left tail, up to a theoretical probability mass of .25, the center of the distribution, from .25 to .75, with a theoretical mass of .5, and the right tail with a theoretical mass of .25. The results are reported in Tables 3-5. The Wald test results confirm that the tails are misspecified across the board for all the densities, although for different reasons. The results do vary somewhat across currencies for the center of the distribution. The conditional test cannot reject that the center of the distribution is well specified for the USD/EUR and JPY/USD.

3. Interval Forecast Evaluation

Berkowitz (2001) has recently argued that it is possible that a density forecast may be rejected overall even if it would provide adequate forecasts for certain segments of the conditional distribution that are of particular interest to the forecaster. In this section we pursue this issue via the construction of interval forecasts from the density forecasts from Section 3. Interval forecasts and the closely related Value-at-Risk forecasts (one-sided interval forecasts) have recently received much interest among financial practitioners as measures of portfolio risk. They are therefore interesting in their own right.

In this section we study the performance of one-month interval forecasts calculated from option prices and FX rates. The interval forecasts are constructed from the one-month option-implied densities which in turn are calculated using the estimation method in Malz (1997) as described in Section 3 above. We have computed conditional interval forecasts for the {0.45, 0.55} probability interval, as well as the {0.35, 0.65}, {0.25, 0.75}, {0.15, 0.85}, and the {0.05, 0.95} intervals. We now set out to evaluate the accuracy of the interval forecasts. To this end consider the following simple framework based on Christoffersen (1998). Let the generic interval forecast be defined as

$$\{L_{t,h}(p_L), H_{t,h}(p_H)\}$$

where p_L and p_H are the percentages associated with the lower and upper conditional quantiles making up the interval forecast.

Consider now the indicator variable defined as

$$I_{t,h} = \begin{cases} 0, \text{if } S_{t+h} \in \{L_{t,h}(p_L), H_{t,h}(p_U)\} \\ 1, \text{if not} \end{cases}$$

Then if the interval forecast is correctly calibrated, we must have that

$$\Pr(I_{t,h} = 1 \mid X_t) = 1 - (p_H - p_L) \equiv p$$

where X_t denotes a vector of information variables (and functions thereof) available on day t. If the interval forecast is correctly calibrated then the expected outcome of the future FX rate falling outside the predicted interval must be a constant equal to the pre-specified interval probability p.

This hypothesis will be tested in a logit regression setup. Under the alternative hypothesis we have

$$\Pr(I_{t,h} = 1 \mid X_t) = \frac{\exp(a + bX_t)}{1 + \exp(a + bX_t)}$$

and the null hypothesis corresponds to the restrictions

$$b = 0,\ a = \ln(p/(1-p)).$$

Running these logit regressions on daily data we again have to worry about overlapping observations, which we allow for using GMM estimation.

Table 6 shows the results for the logit regression tests of the interval forecasts. The interval forecasts for the {0.45, 0.55}, {0.35, 0.65}, {0.25, 0.75}, {0.15, 0.85}, and the {0.05, 0.95} intervals are denoted by the probability of an observation outside the interval, i.e. p=.90, .70, .50, .30 and .10 respectively. We refer to these outside observations as hits. The zero/one hit sequence is regressed on a constant, the 21-day lagged hit and the 21-day lagged 1-month implied volatility. The lagged hit is included to capture any dependence in the outside observations. The implied volatility is included to assess if it is incorporated optimally in the construction of the interval forecast. If the interval forecast is correctly specified then the intercept should be ln(p/(1-p)) and slopes should all be equal to zero. Table 6 reports coefficient estimates along with *t*-

statistics again calculated using GMM with a quadratic kernel and a 21-day bandwidth. Rather than reporting the constant term a, we report a'=a - ln(p/(1-p)) and the t-stat for the test that a'=0. Below the solid line in each subsection of the table the average hit rate, which should be equal to p, is reported along with the t-statistic from the test that the average hit rate indeed equals p. All t-statistics larger than two in absolute value are denoted in boldface type. We also include Wald tests of the joint hypothesis that the intercept is ln(p/(1-p)) and that all the estimated coefficients are zero.

The results in Table 6 can be summarized as follows. First, for the pound the average hit rate is significantly different from the pre-specified p for all but the narrowest interval (with outside probability equal to .90). Second, for the other three FX rates, the average hit rate is typically not significantly different from the pre-specified p. The only notable exception is the wide-range intervals (with outside probability .10) where all but the JPY/EUR intervals are rejected. It thus appears that the interval forecast have the hardest time forecasting the tails of the spot rate distribution.

Third, notice that no regression slopes are significant in the JPY/EUR case. No dependence in the hit sequence is apparent and the information in implied volatilities seems to be used optimally in this case. Fourth, while the interval forecasts for the JPY/EUR are well specified, the intervals for the other three forecasts are typically rejected. The slope on the 21-days lagged implied volatility is most often found to be significantly negative. This indicates that the hits tend to occur when the implied volatility was relatively low on the day the forecast was made. If the intervals had been using the implied volatility information optimally then no dependence should be found between the current implied volatility and the subsequent realization of the hit sequence.

In summary, we find that the option-implied densities apparently have trouble capturing the tail behaviour of the spot rate distributions. The rejection of widest intervals and thus misspecification of the tails of the density forecasts should perhaps not come as a surprise. The density tails are estimated on the basis of an extrapolation of the volatility smile from the values for which option price information is available (that is for deltas equal to .25, .50, and .75).

4. Conclusion and Directions for Future Work

We have presented evidence on the value of the information in over-the-counter currency option for forecasting various aspects of the distribution of exchange rate movements. We focused on three aspects of spot rate forecasting, namely, volatility, density, and interval forecasting. While other papers have pursued volatility forecasting in manners similar to ours we believe to be the first to systematically investigate the properties of option-based interval and density forecasts.

Our other findings can be summarized as follows. First, the implied volatilities from currency options typically offer predictions that are unbiased and that explain more of the variation in realized volatility than do volatility forecasts based on historical returns only. Second, when evaluating the entire implied density forecasts these are generally rejected. Tests of subsets of the density range suggest that the tails in the distribution are often misspecified. Third, the option-implied intervals are accurate for the JPY/EUR but rejected for the other three currencies in the study. We thus conclude that the information implied in option pricing is helpful for volatility forecasting and for density and interval forecasting as long as the interest is confined to the middle 70% range of the distribution.

The rejection of the widest intervals and the complete density forecast is of course interesting and warrants further scrutiny. The potential reasons are at least fourfold. First, the option contracts used may not have extreme enough strike prices to be useful for constructing accurate distribution tails. Second, the information in options could be used sub-optimally in the density estimates. Third, we could be rejecting the densities because certain information available at the time of the forecasts is not incorporated in the option prices used to construct the densities, i.e. option market inefficiencies. Fourth, the risk premium considerations, which were abstracted from in this paper could be important enough to reject the risk-neutral density forecasts considered. The misspecification of the mean in the case of the JPY/EUR rate suggests that an omitted risk premium could be the culprit in that case. For the other three currencies, however, the culprit appears to be tail misspecification, which is likely to arise from the lack of information on deep in-the-money and deep out-of-the-money options.

Our results suggest several promising venues for future research. First, policy makers may be interested in assessing speculative pressures on a given exchange rate. The option implied densities can be used in this regard by constructing daily option-implied probabilities of say a 3% appreciation or depreciation during the next month. Second, the accuracy of the left and right tail interval forecast could be analyzed separately in order to gain further insight on the probability of a sizable appreciation or depreciation. Third, relying on the triangular arbitrage condition linking the JPY/EUR, the USD/EUR, and the JPY/USD, one can construct option implied covariances and correlations from the option implied volatilities. These implied covariances can then be used to forecast realized covariances as done for volatilities in this paper. Fourth, the misspecification found in the option-implied density forecasts may be rectified by assuming different tail-shapes in the density estimation or by incorporating return-based information. Converting the risk-neutral densities to their statistical counterparts may improve the forecasts as well but will require further assumptions, which may or may not be empirically valid.

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Table 1: MSE and Mincer-Zarnowitz percentage decomposition

	1-Month				3-Month			
USD	MSE	$Bias^2$	Inefficiency	Random	MSE	$Bias^2$	Inefficiency	Random
Implied	7.99	1.25	3.16	95.58	6.46	1.43	5.80	92.77
Historical	12.08	0.00	27.19	72.81	9.16	0.05	28.80	71.15
RiskMetrics	10.47	0.12	18.18	81.69	9.34	0.00	33.43	66.57
GARCH	9.04	2.80	1.96	95.24	6.34	1.24	1.91	96.85

JPY	MSE	$Bias^2$	Inefficiency	Random	MSE	$Bias^2$	Inefficiency	Random
Implied	14.42	1.29	0.76	97.94	11.20	0.49	0.51	99.01
Historical	19.76	0.01	21.70	78.29	14.22	0.08	19.53	80.38
RiskMetrics	17.82	0.24	14.83	84.93	15.33	0.00	25.32	74.68
GARCH	17.51	0.01	12.54	87.45	15.98	1.81	21.86	76.33

GBP	MSE	$Bias^2$	Inefficiency	Random	MSE	$Bias^2$	Inefficiency	Random
Implied	6.09	2.45	5.38	92.16	5.92	0.29	14.78	84.93
Historical	9.25	0.00	26.83	73.17	7.98	0.00	33.01	66.98
RiskMetrics	8.47	0.08	20.15	79.78	8.39	0.06	37.81	62.12
GARCH	7.66	0.02	13.68	86.30	7.29	1.11	26.70	72.19

JPY/USD	MSE	$Bias^2$	Inefficiency	Random	MSE	$Bias^2$	Inefficiency	Random
Implied	14.89	2.31	0.93	96.75	12.19	2.56	1.67	95.77
Historical	19.78	0.00	22.96	77.04	16.25	0.02	24.53	75.45
RiskMetrics	17.81	0.15	16.17	83.68	16.61	0.02	28.75	71.23
GARCH	15.66	1.56	1.21	97.23	12.78	2.72	0.55	96.73

Notes to table: For each exchange rate and forecast horizon we regress realized volatility on each volatility forecast and compute the mean squared error (MSE) from the regression. The table reports the MSE along with the percentage Mincer-Zarnowitz decomposition into squared bias, inefficiency and random variation.

Table 2.A: Unconditional Test of Normal Transform

	USD		JPY		GBP		JPY/USD	
	Estimate	t-stat	Estimate	t-stat	Estimate	t-stat	Estimate	t-stat
Mean	0.072	1.018	-0.297	**-4.031**	-0.024	-0.278	-0.040	-0.525
Var	-0.201	**-3.284**	-0.070	-0.809	0.343	**2.244**	-0.073	-0.838
Skew	0.163	0.732	-0.033	-0.120	0.490	1.511	-0.359	-1.243
Kurt	-0.299	-0.727	0.180	0.297	1.153	1.247	0.031	0.043
	Stats	p-val	Stats	p-val	Stats	p-val	Stats	p-val
Wald-test	50.6	0.000	64.0	0.000	29.6	0.000	7.5	0.112

Table 2.B: Conditional Test of Normal Transform

	USD		JPY		GBP		JPY/USD	
	Estimate	t-stat	Estimate	t-stat	Estimate	t-stat	Estimate	t-stat
Const	0.621	**1.962**	-0.346	-1.271	0.780	**2.762**	-0.088	-0.307
Lag LHS	0.128	**2.295**	0.145	**2.573**	0.304	**4.328**	0.120	1.911
1MIV(-21)	-0.050	-1.861	0.010	0.448	-0.095	**-3.115**	0.006	0.247
Const	0.030	0.197	0.128	0.726	0.854	**3.635**	0.160	0.929
Lag LHS	-0.122	**-3.540**	-0.023	-0.463	0.332	**3.150**	-0.011	-0.266
$1MIV(-21)^2$	-0.002	**-2.046**	-0.001	-1.154	-0.009	**-4.003**	-0.002	-1.751
Const	0.605	1.563	-0.239	-0.680	0.860	**2.083**	-0.365	-0.956
Lag LHS	0.021	0.911	0.093	**2.185**	0.328	**2.665**	0.068	**2.239**
$1MIV(-21)^3$	0.000	-1.674	0.000	1.069	-0.001	**-2.258**	0.000	0.413
Const	0.165	0.279	0.334	0.610	1.675	1.861	0.170	0.214
Lag LHS	-0.047	**-2.084**	-0.001	-0.048	0.312	**2.152**	-0.014	-0.833
$1MIV(-21)^4$	0.000	-1.815	0.000	-0.541	0.000	**-2.710**	0.000	-1.208
	Stats	p-val	Stats	p-val	Stats	p-val	Stats	p-val
Wald-test	106.6	0.000	157.7	0.000	118.4	0.000	50.7	0.000

Notes to table: For each exchange rate density forecast we construct the normal transform variable and test the first four unconditional moments in Panel A. In Panel B we regress powers of the normal transform variable on its lag and powers of implied volatility to test the conditional moments. The joint moment hypotheses are assessed in the Wald tests.

Table 3.A: Unconditional Test of the Normal Transform for {a,b} = {0,.25}

	USD		JPY		GBP		JPY/USD	
	Estimate	t-stat	Estimate	t-stat	Estimate	t-stat	Estimate	t-stat
Mean	0.301	**3.831**	-0.036	-0.421	-0.025	-0.262	0.015	0.140
Var	-0.292	**-4.780**	0.055	0.475	0.089	0.629	0.069	0.413
Skew	0.143	0.522	-0.219	-0.714	-0.451	-1.229	-0.359	-0.813
Kurt	-0.235	-0.566	0.345	0.508	0.401	0.473	0.558	0.543
	Estimate	p-val	Estimate	p-val	Estimate	p-val	Estimate	p-val
Coverage	0.217	0.000	0.357	0.000	0.287	0.000	0.264	0.000
	Stats	p-val	Stats	p-val	Stats	p-val	Stats	p-val
Wald-test	723.0	0.000	156.1	0.000	288.6	0.000	262.7	0.000

Table 3.B: Conditional Test of the Normal Transform for {a,b} = {0,.25}

	USD		JPY		GBP		JPY/USD	
	Estimate	t-stat	Estimate	t-stat	Estimate	t-stat	Estimate	t-stat
Const	-0.351	-0.801	-0.578	-1.074	-0.692	-1.270	0.581	0.657
Lag LHS	-0.144	-1.574	-0.035	-0.415	0.242	**3.204**	0.003	0.026
1MIV(-21)	0.083	**2.153**	0.045	1.030	0.053	0.961	-0.052	-0.576
Const	-0.484	**-2.445**	0.679	1.542	0.941	1.116	-0.763	**-1.968**
Lag LHS	-0.164	-1.259	-0.012	-0.164	-0.009	-0.115	-0.011	-0.171
$1MIV(-21)^2$	0.002	0.968	-0.005	**-2.029**	-0.008	-1.059	0.006	**2.040**
Const	-0.057	-0.103	-1.121	-1.170	-2.166	-1.304	-0.175	-0.205
Lag LHS	0.033	0.219	-0.061	-1.323	0.060	0.914	-0.030	-0.761
$1MIV(-21)^3$	0.001	**3.140**	0.000	1.145	0.001	0.995	0.000	-0.306
Const	-0.249	-0.309	2.092	1.154	4.185	1.164	-1.189	-0.785
Lag LHS	-0.002	-0.015	-0.029	-0.768	-0.038	-0.691	-0.027	-0.783
$1MIV(-21)^4$	0.000	1.541	0.000	**-2.031**	0.000	-1.328	0.000	1.340
	Estimate	p-val	Estimate	p-val	Estimate	p-val	Estimate	p-val
Coverage	0.217	0.000	0.357	0.000	0.287	0.000	0.264	0.000
	Stats	p-val	Stats	p-val	Stats	p-val	Stats	p-val
Wald-test	694.1	0.000	221.5	0.000	370.0	0.000	1019.7	0.000

Notes to table: For each exchange rate density forecast we construct the normal transform variable for the left tail and test the first four unconditional moments in Panel A. In Panel B we regress powers of the normal transform variable on its lag and powers of implied volatility to test the conditional moments.

Table 4.A: Unconditional Test of the Normal Transform for {a,b} = {.25,.75}

	USD		JPY		GBP		JPY/USD	
	Estimate	t-stat	Estimate	t-stat	Estimate	t-stat	Estimate	t-stat
Mean	-0.055	-0.979	-0.137	**-2.375**	-0.189	**-2.934**	0.024	0.413
Var	-0.066	-1.445	0.050	0.900	0.034	0.535	0.072	1.314
Skew	0.142	0.801	0.188	1.139	0.061	0.340	0.058	0.357
Kurt	0.223	0.580	0.061	0.190	-0.031	-0.094	-0.102	-0.339
	Estimate	p-val	Estimate	p-val	Estimate	p-val	Estimate	p-val
Coverage	0.515	0.561	0.477	0.379	0.453	0.082	0.475	0.319
	Stats	p-val	Stats	p-val	Stats	p-val	Stats	p-val
Wald-test	20.7	0.001	87.1	0.000	82.9	0.000	13.8	0.017

Table 4.B: Conditional Test of the Normal Transform for {a,b} = {0.25,.75}

	USD		JPY		GBP		JPY/USD	
	Estimate	t-stat	Estimate	t-stat	Estimate	t-stat	Estimate	t-stat
Const	0.080	0.201	0.315	1.182	0.499	1.292	0.374	1.361
Lag LHS	-0.005	-0.098	-0.063	-0.924	0.050	0.906	-0.093	-1.598
1MIV(-21)	-0.012	-0.377	-0.036	-1.623	-0.098	**-2.350**	-0.026	-1.235
Const	0.004	0.023	-0.322	**-2.130**	0.350	1.910	0.280	1.866
Lag LHS	-0.033	-1.237	0.053	0.889	0.001	0.022	0.060	1.435
$1MIV(-21)^2$	0.000	-0.394	0.003	**2.713**	-0.002	-1.487	-0.001	-1.204
Const	0.279	0.714	0.613	**2.169**	0.100	0.241	0.269	1.012
Lag LHS	0.010	0.428	-0.009	-0.126	0.033	0.728	-0.041	-0.924
$1MIV(-21)^3$	0.000	-0.875	0.000	-1.655	-0.001	-1.683	0.000	-1.615
Const	0.010	0.018	-0.530	-0.962	0.826	1.443	0.440	0.859
Lag LHS	-0.031	**-2.251**	0.072	0.933	-0.010	-0.271	0.014	0.764
$1MIV(-21)^4$	0.000	0.074	0.000	1.785	0.000	-1.862	0.000	-1.165
	Estimate	p-val	Estimate	p-val	Estimate	p-val	Estimate	p-val
Coverage	0.515	0.561	0.477	0.379	0.453	0.082	0.475	0.319
	Stats	p-val	Stats	p-val	Stats	p-val	Stats	p-val
Wald-test	20.7	0.078	49.1	0.000	92.3	0.000	15.0	0.305

Notes to table: For each exchange rate density forecast we construct the normal transform variable for the center of the distribution and test the first four unconditional moments in Panel A. In Panel B we regress powers of the normal transform variable on its lag and powers of implied volatility to test the conditional moments.

Table 5.A: Unconditional Test of the Normal Transform for {a,b} = {.75,1}

	USD		JPY		GBP		JPY/USD	
	Estimate	t-stat	Estimate	t-stat	Estimate	t-stat	Estimate	t-stat
Mean	-0.075	-0.978	-0.212	-1.893	0.256	1.844	-0.280	**-3.476**
Var	-0.205	**-2.610**	-0.231	-1.755	0.637	**2.504**	-0.309	**-4.462**
Skew	0.010	0.036	0.403	0.866	0.729	1.791	-0.213	-0.633
Kurt	0.306	0.462	-0.129	-0.125	0.663	0.761	0.232	0.394
	Estimate	p-val	Estimate	p-val	Estimate	p-val	Estimate	p-val
Coverage	0.268	0.000	0.166	0.000	0.260	0.000	0.261	0.000
	Stats	p-val	Stats	p-val	Stats	p-val	Stats	p-val
Wald-test	316.7	0.000	730.0	0.000	316.9	0.000	664.4	0.000

Table 5.B: Conditional Test of the Normal Transform for {a,b} = {0.75,1}

	USD		JPY		GBP		JPY/USD	
	Estimate	t-stat	Estimate	t-stat	Estimate	t-stat	Estimate	t-stat
Const	0.455	0.845	-1.667	-1.818	1.245	**2.968**	0.205	0.258
Lag LHS	-0.221	**-2.937**	0.031	0.163	0.377	**2.264**	0.103	1.021
1MIV(-21)	-0.051	-0.931	0.149	1.608	-0.122	**-2.078**	-0.022	-0.275
Const	-0.578	**-3.380**	-1.035	-1.845	1.171	**2.377**	-0.394	-0.891
Lag LHS	0.010	0.193	0.018	0.150	0.298	1.603	-0.137	-1.674
$1MIV(-21)^2$	0.003	**2.281**	0.010	1.690	-0.003	-0.556	0.003	0.827
Const	0.127	0.233	-2.203	-1.790	1.989	**2.977**	0.338	0.247
Lag LHS	-0.072	-0.982	-0.093	-0.623	0.431	**2.067**	0.000	-0.005
$1MIV(-21)^3$	0.000	-0.515	0.002	1.870	-0.001	**-2.054**	0.001	0.426
Const	-0.307	-0.332	-4.195	**-2.108**	3.140	**2.147**	0.228	0.127
Lag LHS	0.018	0.726	-0.038	-0.267	0.331	1.396	-0.104	-1.693
$1MIV(-21)^4$	0.000	0.969	0.000	1.908	0.000	-1.885	0.000	1.122
	Estimate	p-val	Estimate	p-val	Estimate	p-val	Estimate	p-val
Coverage	0.268	0.000	0.166	0.000	0.260	0.000	0.261	0.000
	Stats	p-val	Stats	p-val	Stats	p-val	Stats	p-val
Wald-test	446.3	0.000	723.4	0.000	328.3	0.000	504.7	0.000

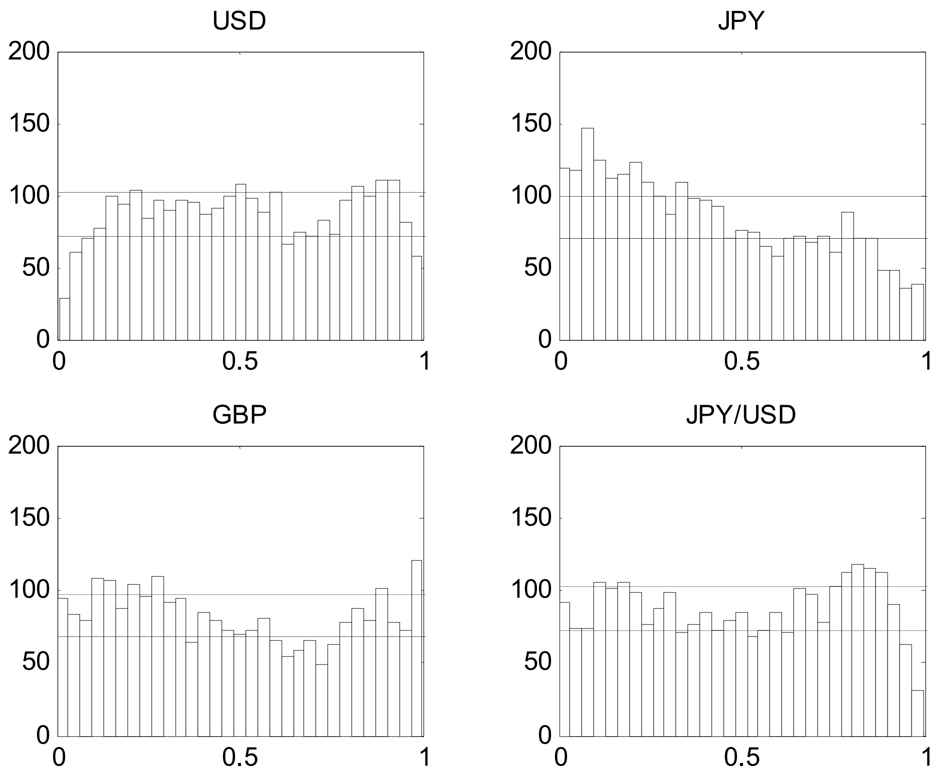
Notes to table: For each exchange rate density forecast we construct the normal transform variable for the right tail and test the first four unconditional moments in Panel A. In Panel B we regress powers of the normal transform variable on its lag and powers of implied volatility to test the conditional moments.

Table 6: Logit Regressions for the Interval Forecasts

	USD		JPY		GBP		JPY/USD	
p = .90	Estimate	t-stat	Estimate	t-stat	Estimate	t-stat	Estimate	t-stat
a'	-0.141	-1.500	0.113	1.042	0.178	1.426	0.158	1.531
Lag hit	-0.050	-0.201	-0.057	-0.221	0.328	1.349	-0.210	-0.689
1 month IV	-0.048	-1.412	0.022	0.621	-0.133	**-3.059**	-0.090	**-2.567**
Average Hit	0.887	-1.362	0.911	1.259	0.912	1.231	0.912	1.389
	Stats	p-val	Stats	p-val	Stats	p-val	Stats	p-val
Wald Test	4.937	0.177	1.763	0.623	12.922	0.005	9.043	0.029
p=.70	Estimate	t-stat	Estimate	t-stat	Estimate	t-stat	Estimate	t-stat
a'	-0.083	-0.884	0.126	1.277	0.327	**2.961**	0.161	1.649
Lag hit	-0.363	**-2.308**	-0.075	-0.471	-0.106	-0.650	-0.030	-0.198
1 month IV	-0.101	**-2.701**	-0.011	-0.342	-0.183	**-4.450**	-0.103	**-2.694**
Average Hit	0.681	-0.932	0.724	1.227	0.755	**2.689**	0.727	1.382
	Stats	p-val	Stats	p-val	Stats	p-val	Stats	p-val
Wald Test	12.723	0.005	1.777	0.620	25.702	0.000	10.614	0.014
p=.50	Estimate	t-stat	Estimate	t-stat	Estimate	t-stat	Estimate	t-stat
a'	-0.022	-0.212	0.147	1.379	0.293	**2.661**	0.108	1.095
Lag hit	-0.313	**-2.058**	-0.088	-0.589	0.049	0.326	-0.019	-0.126
1 month IV	-0.106	**-2.403**	-0.026	-0.771	-0.230	**-5.099**	-0.094	**-2.508**
Average Hit	0.494	-0.232	0.534	1.283	0.570	**2.521**	0.526	1.020
	Stats	p-val	Stats	p-val	Stats	p-val	Stats	p-val
Wald Test	10.291	0.016	2.456	0.483	30.645	0.000	8.176	0.043
p=.30	Estimate	t-stat	Estimate	t-stat	Estimate	t-stat	Estimate	t-stat
a'	-0.171	-1.368	0.038	0.296	0.243	1.997	-0.121	-0.926
Lag hit	-0.491	**-2.560**	-0.127	-0.628	0.662	**3.384**	-0.170	-0.774
1 month IV	-0.149	**-2.501**	-0.055	-1.282	-0.253	**-4.772**	-0.125	**-2.502**
Average Hit	0.271	-1.196	0.306	0.234	0.370	**2.293**	0.284	-0.617
	Stats	p-val	Stats	p-val	Stats	p-val	Stats	p-val
Wald Test	15.527	0.001	1.954	0.582	40.743	0.000	6.336	0.096
p=.10	Estimate	t-stat	Estimate	t-stat	Estimate	t-stat	Estimate	t-stat
a'	-0.590	**-2.892**	-0.024	-0.118	0.309	1.685	-0.491	-1.979
Lag hit	-2.611	**-2.474**	0.001	0.002	1.561	**3.919**	-0.609	-0.854
1 month IV	-0.168	-1.913	-0.046	-0.611	-0.296	**-3.782**	-0.081	-0.698
Average Hit	0.066	**-2.628**	0.097	-0.139	0.166	**2.424**	0.065	**-2.411**
	Stats	p-val	Stats	p-val	Stats	p-val	Stats	p-val
Wald Test	14.639	0.002	0.383	0.944	45.168	0.000	4.130	0.248

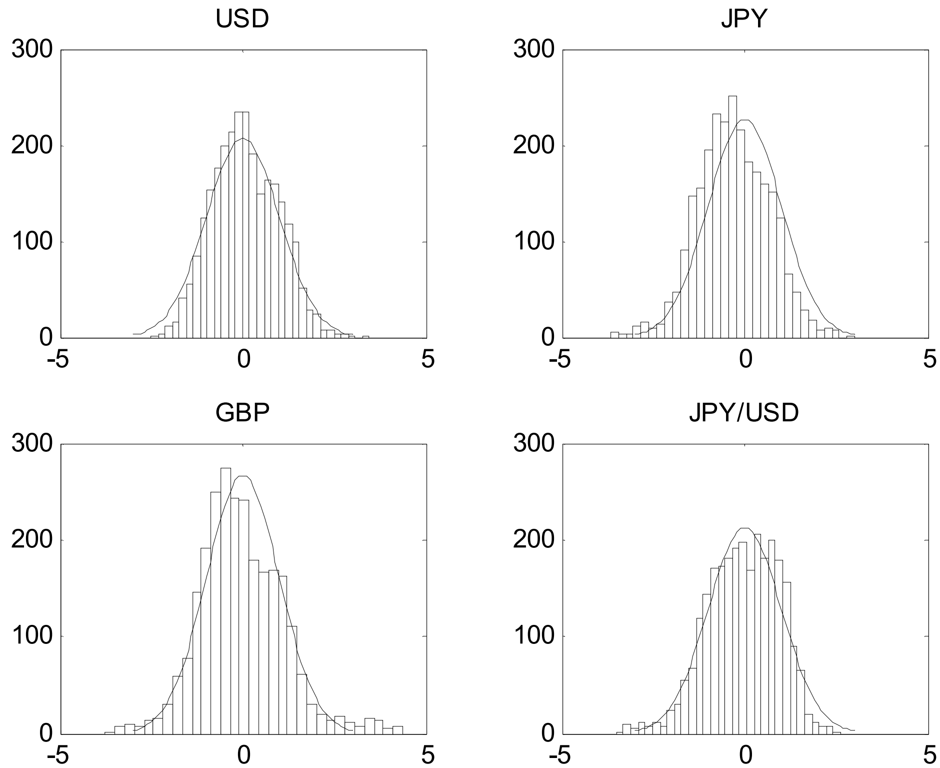
Notes to table: For each exchange rate and each interval coverage rate, p, we run a logit regression on a constant, the lagged hit rate and the 1-month implied volatility. The table reports the estimate of the centred constant term, a’=a - ln(p/(1-p) and the slope coefficients. The average hit rate is also reported as well as a Wald test that a’ and the slope coefficients are all jointly zero.

Figure 1: Histogram of Probability Transforms with 90% Confidence Band



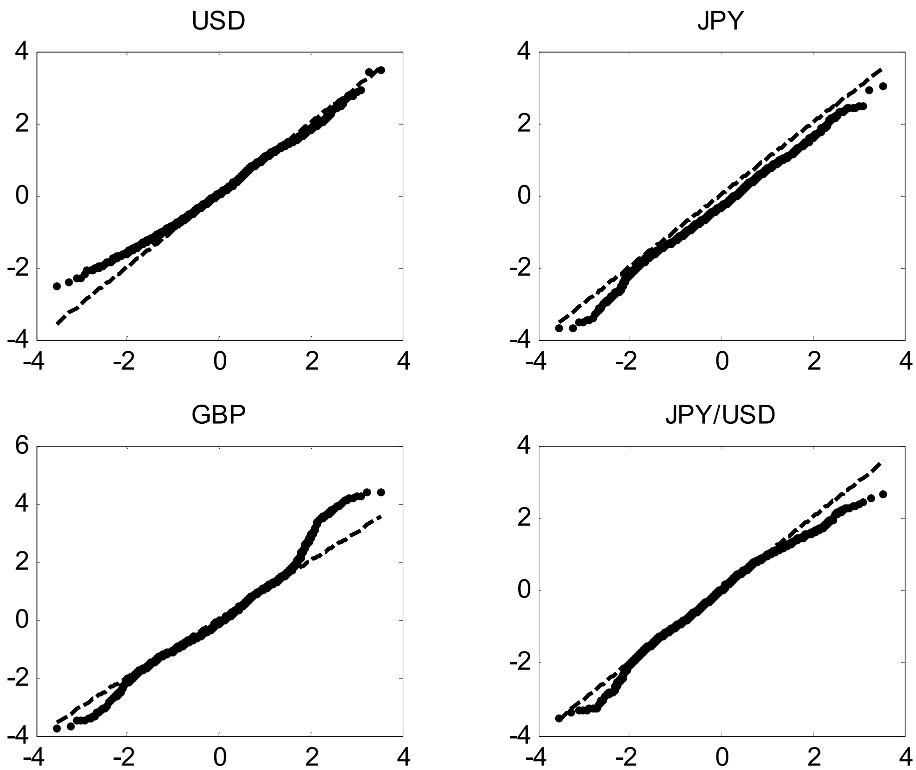
Notes to figure: For each exchange rate we plot the histogram of the probability transform variable from the option implied density forecasts. The horizontal lines denote the 90% confidence interval that the transform variables are uniformly distributed.

Figure 2: Histogram of Normal Transforms with Normal Distribution Imposed



Notes to figure: For each exchange rate we plot the histogram of the normal transform variable from the option implied density forecasts. Each histogram is superimposed on a normal density function.

Figure 3: QQ Plots of Normal Transform Variables



Notes to figure: For each exchange rate we scatter plot the empirical quantile of the normal transform variable from the option implied density forecast against the corresponding quantile of the normal distribution. The diagonal line denotes a perfect fit.

APPENDIX

Table A.1: Foreign Exchange Descriptive statistics. Pre 1999

	Daily Returns				Squared Daily Returns			
	USD/DEM	JPY/DEM	GBP/DEM	JPY/USD	USD/DEM	JPY/DEM	GBP/DEM	JPY/USD
Mean	-8.25E-05	-4.11E-05	2.21E-05	-3.57E-05	5.07E-05	5.35E-05	2.89E-05	6.21E-05
Std. Dev.	7.12E-03	7.32E-03	5.38E-03	7.88E-03	1.03E-04	1.64E-04	8.33E-05	1.96E-04
Skewness	-0.110	-0.876	0.653	-0.855	6.129	11.998	20.852	14.721
Kurtosis	5.084	10.346	9.267	10.962	57.905	194.273	634.957	328.912
Jarque-Bera	306	3783	2836	4615	220218	2465015	27760096	7451371
Observations	1670	1592	1661	1670	1670	1592	1661	1670

	Daily Returns Pairwise Correlations			Squared Daily Returns Pairwise Correlations			
	USD/DEM	JPY/DEM	GBP/DEM		USD/DEM	JPY/DEM	GBP/DEM
JPY/DEM	0.301			JPY/DEM	0.155		
GBP/DEM	0.348	0.087		GBP/DEM	0.124	0.073	
JPY/USD	-0.467	0.228	-0.203	JPY/USD	0.215	0.326	0.045

Table A.2: Foreign Exchange Descriptive statistics. Post 1999

	Daily Returns				Squared Daily Returns			
	USD/EUR	JPY/EUR	GBP/EUR	JPY/USD	USD/EUR	JPY/EUR	GBP/EUR	JPY/USD
Mean	-1.41E-04	-1.18E-04	-1.17E-04	2.24E-05	4.79E-05	7.07E-05	2.54E-05	4.43E-05
Std. Dev.	6.92E-03	8.41E-03	5.04E-03	6.66E-03	8.93E-05	1.60E-04	4.64E-05	8.39E-05
Skewness	0.422	0.046	0.347	-0.149	8.472	8.871	4.300	4.836
Kurtosis	4.512	6.107	4.379	4.581	139.912	128.608	28.362	35.773
Jarque-Bera	129	415	102	111	816790	690618	30778	50109
Observations	1030	1030	1030	1030	1030	1030	1030	1030

	Daily Returns Pairwise Correlations			Squared Daily Returns Pairwise Correlations			
	USD/EUR	JPY/EUR	GBP/EUR		USD/EUR	JPY/EUR	GBP/EUR
JPY/EUR	0.638			JPY/EUR	0.610		
GBP/EUR	0.704	0.487		GBP/EUR	0.421	0.212	
JPY/USD	-0.234	0.600	-0.117	JPY/USD	0.134	0.488	0.011

Notes to tables: Descriptive statistics are calculated for daily FX returns and squared returns. Prior to 1999 exchange rates are quoted against the DEM (except for the JPY/USD) and post 1999 exchange rates are quoted against the EUR.

Table A.3: Foreign Exchange Volatility: Descriptive statistics

USD	1-M IV	3-M IV	1-M HV	3-M HV	1-M GARCH	3-M GARCH	RM
Mean	10.94	11.07	10.69	10.90	11.17	11.06	10.81
Std. Dev.	2.34	1.88	3.36	2.72	2.01	1.60	3.00
Skewness	0.90	0.44	1.29	0.94	1.43	1.29	1.18
Kurtosis	4.59	3.03	5.19	3.76	6.59	6.42	4.82
Jarque-Bera	662.1	87.1	1378.7	497.7	2545.0	2207.6	1072.3
Observations	2748	2748	2893	2893	2893	2893	2893

JPY	1-M IV	3-M IV	1-M HV	3-M HV	1-M GARCH	3-M GARCH	RM
Mean	11.69	11.73	11.29	11.59	11.28	10.92	11.46
Std. Dev.	3.21	2.68	4.71	4.01	4.16	4.07	4.32
Skewness	0.61	0.44	1.16	0.59	1.21	1.02	0.96
Kurtosis	3.87	2.96	4.76	2.83	5.25	4.48	4.39
Jarque-Bera	256.0	88.3	1019.4	169.6	1316.5	769.1	679.7
Observations	2748	2748	2893	2893	2893	2893	2893

GBP	1-M IV	3-M IV	1-M HV	3-M HV	1-M GARCH	3-M GARCH	RM
Mean	8.13	8.04	7.66	7.82	7.73	7.56	7.75
Std. Dev.	2.28	1.91	2.97	2.48	2.45	2.27	2.71
Skewness	0.35	-0.07	1.68	1.03	2.20	2.34	1.42
Kurtosis	3.21	2.85	8.99	4.80	13.35	14.99	7.05
Jarque-Bera	61.9	4.7	5672.6	902.8	15229.3	19959.6	2945.0
Observations	2747	2747	2893	2893	2893	2893	2893

JPY/USD	1-M IV	3-M IV	1-M HV	3-M HV	1-M GARCH	3-M GARCH	RM
Mean	11.47	11.66	10.89	11.13	11.37	11.68	11.04
Std. Dev.	3.00	2.53	4.58	3.90	2.89	2.17	4.20
Skewness	1.26	1.04	2.35	2.12	3.49	3.91	2.31
Kurtosis	6.29	4.39	12.21	9.71	22.43	26.80	12.09
Jarque-Bera	1970.8	713.4	12897.5	7595.6	51381.6	75617.6	12539.0
Observations	2748	2748	2893	2893	2893	2893	2893

Notes to Table: For each exchange rate we compute the descriptive statistics for the various annualized 1-month and 3-month volatility forecasts analyzed in Section 2. The forecasts are: Implied Volatility (IV), Historical Volatility (HV), GARCH, and RiskMetrics (RM). For RiskMetrics the forecast is the same for both horizons. The sample period is March 31, 1992 – February 19, 2003.

Table A.4: Foreign Exchange Volatility Forecasts: Correlation

USD

	1-Month					3-Month			
	IV	RV	HV	GARCH		IV	RV	HV	GARCH
RV	0.554				RV	0.459			
HV	0.790	0.452			HV	0.818	0.376		
GARCH	0.793	0.471	0.915		GARCH	0.723	0.437	0.753	
RM	0.844	0.473	0.945	0.922	RM	0.797	0.424	0.869	0.903

JPY

	1-Month					3-Month			
	IV	RV	HV	GARCH		IV	RV	HV	GARCH
RV	0.609				RV	0.591			
HV	0.821	0.558			HV	0.866	0.572		
GARCH	0.845	0.567	0.943		GARCH	0.798	0.537	0.792	
RM	0.871	0.571	0.952	0.951	RM	0.837	0.571	0.892	0.926

GBP

	1-Month					3-Month			
	IV	RV	HV	GARCH		IV	RV	HV	GARCH
RV	0.585				RV	0.398			
HV	0.812	0.482			HV	0.781	0.362		
GARCH	0.780	0.498	0.915		GARCH	0.699	0.374	0.684	
RM	0.847	0.484	0.956	0.894	RM	0.791	0.390	0.879	0.888

JPY/USD

	1-Month					3-Month			
	IV	RV	HV	GARCH		IV	RV	HV	GARCH
RV	0.569				RV	0.519			
HV	0.793	0.597			HV	0.801	0.617		
GARCH	0.789	0.605	0.908		GARCH	0.691	0.549	0.741	
RM	0.837	0.619	0.960	0.905	RM	0.782	0.619	0.913	0.890

Notes to table: For each exchange rate and forecast horizon we compute the correlation matrix for the four competing volatility forecasts considered in Section 2. The sample period is March 31, 1992 – February 19, 2003.

Footnotes

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[1] The OTC volatilities used in this paper were provided by Citibank N.A

[2] Prior to January 1, 1999 these were denoted in DEM.

[3] See e.g. Baillie and Bollerslev (1989).

[4] The descriptive statistics for the daily FX returns are reported in the appendix tables A.1 and A.2.

[5] It is however not necessarily innocuous when using intraday data to calculate daily realized volatilities as done for example in Andersen, Bollerslev, Diebold and Labys (2003).

[6] The GARCH model contains parameters which must be estimated. We do this on rolling 10-year samples starting in January 1982 and using QMLE. Each year we forecast volatility one-year out-of-sample before updating the estimation sample by another calendar year of daily returns. The euro volatility forecasts are constructed using synthetic euro rates in the period prior to the introduction of the euro.

[7] Table A.3 in the appendix reports descriptive statistics on 1-month and 3-month volatility forecasts from the four forecasting models. Table A.4 reports contemporaneous correlations between the four volatility forecasts at each horizon. Tables A.4 and A.5 cover the entire 1993-2003 sample.

[8] See also Bandi and Perron (2003), Chernov (2003), Bates (2002), and Benzoni (2001).

[9] A strangle consists of a long position in an out-of the-money call and an out-of the-money put. A risk reversal consists of a long position in an out-of-the-money call and a short position in a out-of-the-money put.

[10] The delta refers to the sensitivity of the call option with respect to the underlying exchange rate.

[11] We limit our attention to 30 day density forecasts as our dataset does not include 3-month risk reversal and straddles.

[12] See also Diebold, Gunther and Tay (1998) and Diebold, Hahn and Tay (1999).

[13] The superimposed normal distribution functions have different heights due to the different number of observations available for each currency.

[14] See Bontemps and Meddahi (2005) for related testing procedures.

[15] When implementing the tests we subtract the sample mean from Z in the variance equation and we further divide by the sample standard deviation in the skewness and kurtosis regressions.

[16] Sub-sample tests not reported here reveals that the full sample rejection of the GBP forecasts is largely due to problems early in the sample.