1 Nonparametric Testing of ARCH for Option Pricing

Peter Christoffersen Jinyong Hahn

1.1 Introduction

Derivative securities are widely traded financial instruments which inherit their properties from the underlying assets. The well-known Black and Scholes formula [Black and Scholes 1973] is one of the few cases where the price of a derivative asset (a European call option) can be analytically expressed. Unfortunately, the set of assumptions underlying the Black-Scholes economy are rarely met in practice. Among many possible extensions of the Black-Scholes assumptions, modification of the constant volatility assumption has perhaps received the most attention in the econometric community. The ARCH model developed for the discretely sampled data by [Engle 1982] and its many variants have received a lot of attention, perhaps because they were expected to capture the spirit of continuous time models as in [Hull and White 1987]. This intuition was apparently confirmed by [Nelson 1996] and [Nelson and Foster 1994], who established that the continuous-record asymptotics for some discrete-time ARCH processes converge to the continuous-time processes of [Hull and White 1987].[1]

The purpose of this chapter is to nonparametrically examine the relevance of ARCH models in option pricing. We ignore the predictive aspects of ARCH models and examine relevance solely based on their performance in option pricing. It is interesting to note that much of the ARCH literature verbally emphasizes the potential of ARCH volatility in option pricing, but not many studies statistically test the relation between the ARCH models and option pricing. Lacking statistical testing, a few authors do investigate the link between ARCH models and option pricing, these include [Duan 1995], [Engle and Mustafa 1992], [Engle et al 1996], [Bollerslev and Mikkelsen 1995], and [Amin and Ng 1993]. All the results seem to imply at least two assumptions. The first assumption, which we will call the sufficiency assumption, is that the volatility is a mean-sufficient statistic for the option price of interest given other relevant variables. The second assumption, which we will call the necessity assumption, is that the ARCH volatility has additional explanatory power given other relevant variables. This amounts to the assumption

1. Important competitors to the ARCH models include the discrete-time stochastic-volatility models discussed by [Melino and Turnbull 1990], [Wiggins 1987], [Harvey et al 1994], [Jacquier et al 1994], and [Shephard et al 1998]. Due to the ease of implementing ARCH models, and their corresponding dominance among practitioners we focus attention on ARCH in this study.

that the changing volatility model described by ARCH better explains the option price than the constant volatility model.[2] Recent developments in the nonparametric specification literature allow us to test both assumptions. In this chapter, we test the necessity and sufficiency of ARCH models for European option pricing. We apply the tests of [Aït-Sahalia et al 1994] and [Fan and Li 1996] to examine the relevance of ARCH models in pricing European options. We try to examine the usefulness and relevance of ARCH models in option pricing taking a purely reduced form statistical approach. Such reduced form strategy complements the theoretical developments of [Engle and Mustafa 1992], [Amin and Ng 1993], and [Duan 1995].

Nonparametric specification tests have an interesting practical implication. Recently, [Broadie et al 1996b] and [Broadie et al 1996a] have suggested nonparametric estimation of American option prices. They suggest estimating the pricing functional given a parametrically estimated volatility and other relevant variables. Given their popularity in volatility extraction and prediction, ARCH models are a natural choice in such procedure. We thus believe that our nonparametric testing results form nice complements to results on nonparametric estimation by [Broadie et al 1996b].

1.2 Sufficiency and Necessity

In this section, we elaborate on the sufficiency and necessity arguments introduced in the previous section. We first make a very brief review of European option pricing under changing volatility. In the simple Black-Scholes economy with constant volatility, the European call option price C, on an asset without dividend payments, can be written as

$$C = P(K, S, r, T - t, \sigma). \qquad (1.1)$$

Here, $K, S, r, T - t$, and σ denote the strike price, the underlying asset price, the (constant) risk-free interest rate, the time-to-maturity, and the (constant) volatility. When the volatility is characterized by a continuous-time stochastic-volatility model

2. By constant volatility based option pricing, we do not restrict ourselves to the standard Black-Scholes formula. We interpret constant volatility based option pricing to mean any nonparametric specification of the option price which does not include time-varying volatility as one of the explanatory variables.

and when the volatility is uncorrelated with aggregate consumption, then following [Hull and White 1987], we may write,

$$C = E[P(K, S, r, T - t, \sigma_T) | \mathcal{F}_t]; \quad (1.2)$$

where the (conditional) expectation (given all the information available) at time t is taken with respect to σ_T, the random volatility at maturity. On the other hand, when volatility follows a GARCH process and when the equivalent martingale measure Q is such that the log of the underlying asset price follows a particular random walk, [Duan 1995] establishes that the European call can be priced by

$$C_t = \exp(-r(T - t)) E^Q[\max(S_T - K, 0) | \mathcal{F}_t]: \quad (1.3)$$

Observe that all (1.1), (1.2), and (1.3) are all deterministic relations. We do not believe that any of them will survive a common sense test against a real data set if we strictly impose this deterministic interpretation. We thus freely depart from this deterministic interpretation, and interpret each of them in the stochastic way: we are going to read (1.1), (1.2), and (1.3) as "The conditional expectation of the left hand side of the equation given all the available information is equal to the right hand side."[3]

The stochastic interpretation of (1.1), (1.2), and (1.3) has an interesting implication. Observe that all of them imply that, given all the available information $\mathcal{F}_t$, only five variables $K, S, r, T - t, \sigma$ are relevant for option pricing. We may compactly write it as

$$E[C | \mathcal{F}_t] = E[C | K, S, r, T - t, \sigma]: \quad (1.4)$$

Among the five variables of interest, the first four variables $K, S, r, T - t$ are observed. Only σ is unobserved in the option pricing relationship. This implies that σ is some real-valued mapping whose argument is an infinite-dimensional collection of all the available information. To be more specific, (1.4) implies that there exists a one-dimensional statistic which we write (with some abuse of notation) as $\sigma(\mathcal{F}_t)$ such that

$$E[C | \mathcal{F}_t] = \pi(K, S, r, T - t, \sigma(\mathcal{F}_t)) \quad (1.5)$$

3. Although we do not have any finance-economic justification for this stochastic interpretation, we adopt this convention due to the seeming lack of results in the econometrics of option pricing errors. We refer to Renault (1996) for a recent attempt to resolve the difficulties, although we note that the deterministic relation implies the conditional expectation relation in any case.

for some function ψ. We are now ready to obtain the mean sufficiency and necessity of the statistic $\sigma(\mathcal{F}_t)$ given $(K, S, r, T - t)$.

As for sufficiency, observe that (1.5) implies that the conditional expectation given $(K, S, r, T - t)$ and $\sigma(\mathcal{F}_t)$ is equal to the conditional expectation given the filtration $\mathcal{F}_t$: No component of the filtration $\mathcal{F}_t$ has any additional explanatory power for the option price of interest. In the application, we will choose some sensible variable, say V, in the filtration $\mathcal{F}_t$, and try to examine whether

$$E[C|K, S, r, T - t, \sigma(\mathcal{F}_t)] \neq E[C|K, S, r, T - t, \sigma(\mathcal{F}_t), V]:$$

As for necessity, we observe that the statistic $\sigma(\mathcal{F}_t)$ is useless in nonparametric option pricing if we further have

$$E[C|\mathcal{F}_t] = E[C|K, S, r, T - t];$$

i.e., if the exclusion of $\sigma(\mathcal{F}_t)$ does not entail any loss of information. No statistical procedure is available to test such a general implication. We thus restrict our attention to the following implication of the necessity condition:

$$E[C|K, S, r, T - t] \neq E[C|K, S, r, T - t, \sigma(\mathcal{F}_t)]:$$

Observe that the necessity test does not suffer from the same power problem as the sufficiency test. The necessity simply asserts that $\sigma_t(\theta)$ has additional explanatory power, thus no subjectivity is involved in practice given a parametric volatility model.

With the interpretation of $\sigma(\mathcal{F}_t)$ as just some statistic, consider the stochastic interpretation of (1.1), (1.2), and (1.3) again. Equation (1.1) implies that $\sigma(\mathcal{F}_t)$ is a constant valued function under the constant volatility model. Equations (1.2) and (1.3) imply that $\sigma(\mathcal{F}_t)$ may be expressed using a parametric expression in the cases of continuous-time stochastic-volatility model and discrete-time GARCH model, respectively. In every case, the "statistic" takes the form of the "volatility" of the corresponding econometric model parameterized by θ, say. With some abuse of notation again, we may thus write that $\sigma(\theta_0, \mathcal{F}_t)$ is the necessary and sufficient statistic for the option price of interest given $(K, S, r, T - t)$, where θ_0 denotes the true value of the parameter.

It should be noted that a failure of necessity does not necessarily invalidate a parametric econometric model associated with $\sigma(\theta_0, \mathcal{F}_t)$. For example, we may find that $\sigma(\theta_0, \mathcal{F}_t)$ is nonparametrically unnecessary, but it could be the case that $\sigma(\theta_0, \mathcal{F}_t)$ is constant: Under the Black-Scholes formula, the (constant) volatility is nonparametrically unnecessary. The failure may also be due to a high correlation of

$\sigma(\theta_0; F_t)$ with the other explanatory variables. This may happen if the alternative continuous record asymptotics by [Corradi 1997] is more relevant than that of [Nelson 1996] and [Nelson and Foster 1994]: $\sigma(\theta_0; F_t)$ would be a deterministic function of t, and hence, $T - t$. Therefore, results from the necessity test should be more cautiously interpreted than those from the sufficiency test: a parametric, changing volatility model could be a reasonable model of the volatility process itself, but may turn out to be unnecessary for nonparametric option pricing.

1.3 Nonparametric Specification Tests

In this section, we briefly review the intuition of the nonparametric specification tests developed by [Aït-Sahalia et al 1994] (ABS test) and [Fan and Li 1996], and then discuss the implementation of the tests. Consider the conditional expectation of Y given X, where Y is a one-dimensional random variable and X is a random vector consisting of a p-dimensional random vector W and a q-dimensional random vector V. The null hypothesis that V is does not have any explanatory power given W can be written as $E[Y|W, V] = E[Y|W]$. Letting $M(w) = E[Y|W = w]$ and $m(w, v) = E[Y|(W, V) = (w, v)]$, we can rewrite the hypothesis as

$$\Pr[M(W) = m(W, V)] = 1.$$

The intuition of the test by [Aït-Sahalia et al 1994] is based on the following implication of the null hypothesis:

$$E\left[(M(W) - m(W, V))^2\right] = 0.$$

Now, if we are given a relatively large data set consisting of (Y_i, X_i) $i = 1, \ldots, n$, we may be able to estimate $M(w)$ and $m(w, v)$ by any nonparametric method, say, a kernel regression. Denote the regression estimates by $\hat{M}_\omega(w)$ and $\hat{m}_\omega(w, v)$, respectively, i.e.,

$$\hat{M}_\omega(w) = \frac{\sum_{i=1}^n Y_i K\left(\frac{w - W_i}{\omega}\right)}{\sum_{i=1}^n K\left(\frac{w - W_i}{\omega}\right)}, \qquad \hat{m}_\omega(w, v) = \frac{\sum_{i=1}^n Y_i K\left(\frac{w - W_i}{\omega}, \frac{v - V_i}{\omega}\right)}{\sum_{i=1}^n K\left(\frac{w - W_i}{\omega}, \frac{v - V_i}{\omega}\right)},$$

where ω is a suitably chosen bandwidth. As the sample size gets large, we would expect

$$\Gamma_n = \frac{1}{n} \sum_{i=1}^n \left(\hat{M}_\omega(W_i) - \hat{m}_\omega(W_i, V_i)\right)^2 \to 0 \tag{1.6}$$

under the null. If Γ_n is substantially bigger than zero, it can be interpreted as evidence against the hypothesis.[4] Note that the test statistic is based on kernel regressions which use the same bandwidth, ω, under the null and the alternative. [Fan and Li 1996] develop an alternative test using similar intuition.

To see how these tests can be implemented in the volatility specification test, suppose that a theory suggests that the volatility process is governed by a parametric model $\sigma(\theta; F_t)$ with the true value of the parameter equal to θ_0. For simplicity of notation, write $\sigma_t(\theta) = \sigma(\theta; F_t)$. If this theory is correct, then we should observe option prices such that

$$E[C|F_t] = \Pi(K, S, r, T-t, \sigma_t(\theta_0)),$$

for some function Π. Suppose temporarily that we know the true value θ_0. Then the discussion of necessity and sufficiency in the previous section immediately implies the tests to be conducted. For necessity, we can test whether

$$E[C|K, S, r, T-t, \sigma_t(\theta_0)] = E[C|K, S, r, T-t],$$

and for sufficiency, we can test whether

$$E[C|K, S, r, T-t, \sigma_t(\theta_0)] = E[C|K, S, r, T-t, \sigma_t(\theta_0), V]$$

for any relevant $V \in F_t$.

Without knowledge of the exact value of θ_0, it is natural to rely on an estimated value, for example, MLE. Under the null of correct parametric specification of the volatility process, we can usually obtain a $\sqrt{n}$-consistent estimator, call it $\hat{\theta}$. An interesting and useful aspect of the specification test by [Aït-Sahalia et al 1994] is that the asymptotic distribution of the test statistic under the null is not changed even when we replace θ_0 by a $\sqrt{n}$-consistent estimator.[5] Although [Fan and Li 1996] does not explicitly consider such a case, it is reasonable to expect such a behavior in their test as well.[6] This observation suggests the following strategy for the nonparametric test of relevance of ARCH models for option pricing:

---

4. The test statistic by [Aït-Sahalia et al 1994] differs from the intuitive equation (1.6) in order to accommodate a technicality related to kernel regression: Their test static involves trimming based on the estimated density, as is customary in econometric applications of kernel estimation.

5. See Lemma 4.2 in [Aït-Sahalia et al 1994].

6. The intuition is the same as in [Aït-Sahalia et al 1994]. The rate of convergence of parametric estimation is much faster than in the nonparametric case, and the asymptotic distribution of the test statistic by [Fan and Li 1996] is based on the asymptotic distribution of the kernel estimator whose rate of convergence is much slower than $\sqrt{n}$.

1. Estimate an ARCH model.
2. For sufficiency, choose V in the information set which is expected to have additional explanatory power, and apply the nonparametric specification test where the regressor under the null is $(K, S, r, T-t, \sigma_t(\hat{\theta}))$ and the regressor under the alternative additionally includes V.
3. For necessity, apply the nonparametric specification test where the regressor under the null is $(K, S, r, T-t)$ and the regressor under the alternative additionally includes $\sigma_t(\hat{\theta})$.

It is to be noted that the test by [Aït-Sahalia et al 1994] is originally developed for i.i.d. observations, but it applies to dependent data without modification.[7] It is reasonable to expect the same property from the test by [Fan and Li 1996].[8]

1.4 European Options on the S&P 500 Index

In this section we apply the testing methodology laid out above to European option contracts written on the S&P 500 index.

1.4.1 The Data

We apply the data set from [Aït-Sahlia and Lo 1998], graciously provided to us by the authors. The original data set contains 16,923 pairs of call- and put-option prices (bid-ask averages) on the S&P 500 index recorded between January 1, and December 31, 1993. From this, we eliminate contracts with less than one day-to-maturity, implied volatility greater than 70%, or a price less than \$1/8. This filtering leaves 14,431 observations.

Due to problems of infrequent trading of in-the-money options, the need for simultaneous observations of the underlying index, and unobserved dividend streams from the index, we use the implied futures price of the index as opposed to the actual spot price. Our procedure follows [Aït-Sahlia and Lo 1998]. The spot-futures parity links the two by,

$$F_{t,T} = S_t \exp((r_{t,T} - \delta_{t,T})(T - t));$$

where $T - t$ is the number of days-to-maturity, $r_{t,T}$ is the risk-free rate, and $\delta_{t,T}$ is

7. See Section 4.3.2 in [Aït-Sahalia et al 1994].

8. This is because, as in the test by [Aït-Sahalia et al 1994], the covariations induced by a reasonable amount of dependence in the data are expected to be of smaller order.

Table 1.1
Descriptive Statistics of [Aït-Sahalia and Lo 1998] Data Set

Variable	Mean	Std. Dev.	Min.	Med.	Max.
Call Price	24.23	25.41	0.13	16.68	121.93
Put Price	9.75	12.57	0.13	4.73	102.08
Implied Volatility	11.36	3.29	5.07	10.71	36.43
Implied ATM Volatility	9.37	0.86	6.10	9.36	16.47
Time to Maturity	86.64	72.32	1.00	66.00	350.00
Strike Price	440.80	33.02	350.00	440.00	550.00
Futures Prices	455.42	10.26	428.70	457.82	474.44
Risk Free Rate	3.07	0.08	2.85	3.08	3.21

the (constant) dividend rate between time t and T. The implied futures price given from the call-put parity at-the-money is,

$$\begin{aligned} F_{t,T} &= K^{*} + \exp(r_{t,T}(T - \tau)) \\ &\quad \times (H(S_t, K^{*}, T - t, r_{t,T}, \delta_{t,T}) - G(S_t, K^{*}, T - t, r_{t,T}, \delta_{t,T})); \end{aligned}$$

where $H(\cdot)$ and $G(\cdot)$ are the call and put prices respectively. In order to get reliable futures prices, call and put contracts are used where the strike price, $K = K^{*} \approx S_t$, is close to at-the-money. Now, given the implied at-the-money futures price, $F_{t,T}$, the prices from the illiquid in-the-money call contracts can be calculated as,

$$\begin{aligned} H(S_t, K, T - t, r_{t,T}, \delta_{t,T}) &= G(S_t, K, T - t, r_{t,T}, \delta_{t,T}) \\ &\quad + (F_{t,T} - K)\exp(-r_{t,T}(T - t)); \end{aligned}$$

since a strike price, K, corresponding to an illiquid (liquid) in-the-money call contract, automatically corresponds to a liquid (illiquid) out-of-the-money put contract.

Descriptive statistics of the data set are provided in Table 1.1.

1.4.2 Implementation of the Tests

The nonparametric regression under consideration involves many regressors even under the null: We have at least 5 regressors $(K, S_t, r_{t,T}, T - t, \delta_{t,T})$. Even though the nonparametric specification tests work asymptotically, the asymptotic approximation may be poor in finite samples when the number of regressors is large. We thus make a series of assumptions to reduce the number of regressors. In the end, the specification we test is

$$E\left[\frac{C_{t,T,K}}{F_{t,T}}\,\middle|\,F_t\right] = H\left(\frac{K}{F_{t,T}}, T - t; \sigma_t(\theta)\right)$$

for some nonparametric function H, where $F_{t,T}$ is the implied futures price computed via the put-call parity. The specification can be derived under the assumption that

- The current underlying asset price S_t; and the dividend rate $\delta_{t,T}$ enter into Π only through the implied futures price;
- The function Π is homogeneous of degree one in $(F_{t,T}, K)$; and
- The interest rate, $r_{t,T}$, is constant.

Observe that the first two assumptions are satisfied under the standard Black-Scholes formula. As for the second assumption of homogeneity, [Broadie et al 1996b] derived similar results for American option pricing. The second assumption is important not only because of the dimension reduction implication but also because of the presumed stationarity implication. The test by [Aït-Sahalia et al 1994] applies without modification for a general class of stationary processes.[9] But the test does not easily accommodate the nonstationarity that is expected to be present in the underlying asset price level, S_t. Faced with this challenge, it seems reasonable to follow the standard practice of assuming stationarity of $\frac{C_{t,T,K}}{F_{t,T}}$ and $\frac{K}{F_{t,T}}$. Supporting this assumption, we do not find any significant trends in the two ratios in our data set.

We take our benchmark variance specification to be the EGARCH(1,1) model from [Nelson 1991]. Thus we estimate

$$\ln(\sigma_t^2) = \omega + \alpha\left|\frac{\varepsilon_t}{\sigma_{t-1}}\right| + \gamma\frac{\varepsilon_t}{\sigma_{t-1}} + \beta\ln(\sigma_{t-1}^2).$$

9. See their Assumption 4 and related discussion.

Table 1 2
EGARCH(1,1) Estimation

Parameter	Estimate	Standard error	Robust S.E.
Mean	0268	0195	0202
!	-0398	0074	0131
®	0481	0096	0166
°	-0171	0072	0205
¯	9923	0022	0042

Table 1.3
GARCH(1,1) Estimation

Parameter	Estimate	Standard error	Robust S.E.
Mean	0388	0198	0201
!	0031	0011	0018
®	0179	0032	0058
¯	9760	0038	0067

Estimating EGARCH models on the S&P 500 index returns (ex dividends) yields strongly persistent— close to integrated— specifications.[10] In Table 1 2 we show the estimation results from our benchmark specification.

We will also be employing a symmetric GARCH (1,1) model below. In this case, the estimation results are as in Table 1 3:

Finally, let us turn to the actual hypotheses tested. We carry out the following experiments: Cases 1 through 3 test the sufficiency part of our hypothesis:

² In Case 1 we take the EGARCH (1,1) specification to be the null, and include $\not{c}\, S_{t-1}$ j as an explanatory variable under the alternative hypothesis, testing for a more elaborate lag structure in volatility.

² A variable describing the number of days since the last trading day is sometimes included in the conditional variance specification. [French and Roll 1986] suggest the specification, $!_t = ! + \ln(1 + \pm N_t)$, where N_t is the number of calendar days

10. The underlying stock index data are drawn from the CRSP tapes (series SPINDX). We estimate on daily data from April 1988 through December 1993 which gives 1,500 observations. We note that the stock index prices are recorded at a slightly different time of day than are the options. But, any deterministic intraday volatility pattern should be captured well in the subsequent nonparametric regression. Since we use the stock prices only to obtain a parametric estimate of the conditional variance, this is not of major concern.

since the last trading day. In Case 2, we include N_t under the alternative calling it the "Monday Effect," and retain the standard EGARCH(1,1) specification under the null.

- In Case 3 we test for asymmetry under the alternative using a symmetric GARCH(1,1) specification under the null. It is interesting to note that the standard inference in Table 1.2 implies significant asymmetry in the EGARCH(1,1) specification, whereas the Bollerslev-Wooldridge robust inference rejects asymmetry.

Cases 4a and b test the sufficiency part of our hypothesis[11]:

- In Case 4a, we test the necessity of ARCH using a simple homoskedastic, nonparametric regression including just time-to-maturity and strike price over the futures price under the null. The alternative includes an EGARCH(1,1) volatility measure. Although we call the null 'Homoskedasticity', it is somewhat of a misnomer: As noted above, the null includes the case where the volatility is changing but in a deterministic fashion.
- Case 4b is similar to 4a only it applies a symmetric GARCH(1,1) model under the alternative.
- In Cases 5a and b, we restrict Case 4 further, and take the regressor under the null to be the standard parametric Black-Scholes price with homoskedastic innovations. The alternative here includes the benchmark EGARCH(1,1) and GARCH(1,1) conditional volatility respectively. Case 5 is included to check the parametric Black-Scholes specification, but also to check the power of ABS tests in the actual sample at hand.

1.4.3 Test Results

Both test statistics are asymptotically standard normal: the null is rejected in favor of the alternative under 5% significance level if the test statistic exceeds 1.64. To compute the test statistics, we use a Gaussian kernel, where the bandwidth is chosen via cross-validation on a grid of possible values under the alternative, using standardized regressors. The results of the various experiments using the ABS test are summarized in Table 1.4.

Observe that the ARCH model of our choice passes the test of mean sufficiency: In Cases 1, 2, and 3, the nulls which only include the ARCH volatilities cannot be rejected under 5% significance level. Furthermore, the asymmetry does not seem

---

11. In case 1 through 4, the regression under the null includes the time-to-maturity as well as the strike price normalized by the futures price.

Table 1.4
Test Results

Case	Null	Alternative	p + q	ω	ABS test
1	EGARCH(1,1)	$\lvert \Delta S_{t-1} \rvert$	3+1	1.28	-25.4
2	EGARCH(1,1)	Monday Effect	3+1	0.94	-26.2
3	GARCH(1,1)	ΔS_t	3+1	1.25	-25.7
4a	Homoskedasticity	EGARCH(1,1)	2+1	0.61	-12.1
4b	Homoskedasticity	GARCH(1,1)	2+1	0.54	-5.5
5a	Black-Scholes	EGARCH(1,1)	1+1	0.52	77.0
5b	Black-Scholes	GARCH(1,1)	1+1	0.41	90.8

to matter for options pricing. On the other hand, the ARCH model does not pass the test of necessity. In Cases 4a and b, we see that the nonparametric model which does not include any ARCH volatility is accepted. As expected, we get a resounding rejection of the parametric Black-Scholes pricing formula in cases 5a and 5b. We take this result as evidence that the ABS test is quite powerful in the sample at hand.

The reduced-form nature of our approach refrains us from speculation as to the financial economic reason why the ARCH models do not pass the test of necessity. We conjecture that the volatility process has a high correlation with the other regressors included in the null, so that it does not have any additional explanatory power over and above the homoskedastic, nonparametric regression. Another possible explanation, namely sensitivity of our results with respect to bandwidth choice is explored in the next section. Finally, we remark that the application of the test by [Fan and Li 1996] yielded similar results.[12]

1.4.4 Robustness of Test Results to the Choice of Bandwidth

In order to avoid arbitrariness in the choice of bandwidth, every experiment above builds on a cross-validated bandwidth parameter. To complement this data-based bandwidth choice, we conducted some sensitivity analysis of the test results in Table 1.4 to the choice of bandwidth. First, consider the sufficiency test of GARCH, i.e.

12. We again used the Gaussian kernel in this application. [Fan and Li 1996] test requires that the bandwidth a under the null be different the bandwidth h under the alternative: We need $a/h \to 1$ asymptotically. Because we would have $a/h \to 0$ if we chose both a and h by crossvalidation, we chose h by cross validation and experimented with $a = h$; $5h$; and $10h$. Although it is possible that our bandwidth choice was misleading, we find that the power of [Fan and Li 1996] rather low: We accepted the nulls for every case we considered, including the null of parametric Black-Scholes price.

cases 1-3. In neither of the three experiments conducted, did we reject the null hypothesis of sufficiency for any reasonable bandwidth choice. Thus the sufficiency results seem robust.

Second, consider the necessity tests of GARCH in case 4a and b. The rejection of the neccesity of GARCH is somewhat sensitive to the choice of bandwidth. For a bandwidth markedly different from the cross-validated one, namely larger than 0.1, the test rejects the null that the nonparametric Black-Scholes model is sufficient, and thus concludes that GARCH or EGARCH is necessary for options pricing.

Finally, the rejection of Black-Scholes in case 5a and b in Table 1.4 is robust to any reasonable choice of bandwith.

1.5 Summary and Concluding Remarks

We have considered the relevance of ARCH models in nonparametric option pricing. Relevance was decomposed into two components: sufficiency and necessity. Using the tests by [Aït-Sahalia et al 1994] and [Fan and Li 1996], we conclude that ARCH models do pass the test of sufficiency. As for the test of necessity, we find the test result to be somewhat sensitive to the bandwidth choice, although ARCH volatility fails to pass the necessity test in a fairly large region around the cross-validated bandwidth. We thus tentatively conclude that ARCH models might be irrelevant for European option pricing in a nonparametric statistical sense.

An ever open question in statistical hypothesis testing is the power of the applied tests in the particular situation under investigation. While we have a reasonably large sample at hand, the power of the nonparametric tests applied might not be as high as would be desired. Some rigorous Monte Carlo experiments or higher order expansions are necessary to validate or invalidate this concern. While this is beyond the scope of the present chapter, we plan to address it in future work.

Despite our conclusions, market participants and academics might still be interested in ARCH models since the nonparametric option pricing relationship in population cannot be hurt by the inclusion of additional variables. ARCH models may be interesting in practice solely in an economic sense. This tentative conclusion is supported by the fact that ARCH models pass the test of sufficiency, and that they do offer significant improvement over parametric Black-Scholes prices.

References

Aït-Sahalia, Y., P. Bickel, and T. Stoker. 1994. Goodness-of-fit tests for regression using kernel methods. unpublished manuscript.

Aït-Sahlia, Y. and A. Lo 1998. Nonparametric estimation of state-price-densities implicit in financial asset prices. Journal of Finance, pages 499{547.

Amin, K. and K. Ng. 1993. Option valuation with systematic stochastic volatility. Journal of Finance, 48:881{910.

Black, F. and M. Scholes. 1973. The pricing of options and corporate liabilities. Journal of Political Economy, 81:637{659.

Bollerslev, T. and H. Mikkelsen. 1995. Long-term equity anticipation securities and stock market volatility dynamics. Unpublished Manuscript.

Broadie, M., J. Detemple, E. Ghysels, and O. Torrès. 1996a. American options with stochastic dividends and volatility: A nonparametric investigation. Unpublished Manuscript.

Broadie, M., J. Detemple, E. Ghysels, and O. Torrès. 1996b. Nonparametric estimation of american options exercise boundaries and call prices. Unpublished Manuscript.

Corradi, V. 1997. Degenerate continuous time limits for GARCH and GARCH-type processes. Unpublished Manuscript.

Duan, J. 1995. The GARCH option pricing model. Mathematical Finance, 5:13{32.

Engle, R. 1982. Autoregressive conditional heteroscedasticity with estimates of the variance of UK inflation. Econometrica, 50:987{1008.

Engle, R., A. Kane, and J. Noh. 1996. Index-option pricing with stochastic volatility and the value of accurate variance forecasts. Review of Derivative Research, 1:139{157.

Engle, R. and C. Mustafa. 1992. Implied ARCH models from option prices. Journal of Econometrics, 52:289{311.

Fan, Y. and Q. Li. 1996. Consistent model specification tests: Omitted variables and semiparametric functional forms. Econometrica, 64:865{890.

French, K. and R. Roll. 1986. Stock return variances: The arrival of information and the reaction of traders. Journal of Financial Economics, 17:5{26.

Harvey, A., E. Ruiz, and N. Shephard. 1994. Multivariate stochastic variance models. Review of Economic Studies, 61:247{264.

Hull, J. and A. White. 1987. The pricing of options on assets with stochastic volatilities. Journal of Finance, 42:281{300.

Jacquier, E., N. Polson, and P. Rossi. 1994. Bayesian analysis of stochastic volatility models. Journal of Business and Economic Statistics, 12:371{389.

Melino, A. and S. Turnbull. 1990. Pricing foreign currency options with stochastic volatility. Journal of Econometrics, 45:239{265.

Nelson, D. 1991. Conditional heteroskedasticity in asset returns: A new approach. Econometrica, 59:347{370.

Nelson, D. 1996. Asymptotically optimal smoothing with ARCH models. Econometrica, 64:561{573.

Nelson, D. and D. Foster. 1994. Asymptotic filtering theory for univariate ARCH models. Econometrica, 62:1{41.

Shephard, N., S. Kim, and S. Chib. 1998. Stochastic volatility: Likelihood inference and comparison with ARCh models. Review of Economic Studies, 65:361{393.

Wiggins, J. 1987. Option values under stochastic volatility: Theory and empirical estimates. Journal of Financial Economics, 19:351{372.